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MINIMIZATION OF SURFACE ROUGHNESS OF MILD STEEL (EN10) MATERIAL USING A MODIFIED MODEL ROBUST REGRESSION 2

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ABSTRACT

For a machined mild steel material of certain physical and chemical properties, the surface roughness, prescribed in four different parameters each of which is desired to be as minimal as possible, is known to be a function of the machining (cutting) parameters including cutting speed (m/min), feed rate (mm/rev) and depth of cut (mm).This functional relationship between each of the roughness parameters and machining parameters must be represented by a model and subsequently optimized to determine the optimal values of the machining parameters that minimize the parameters of surface roughness. In Response Surface Methodology (RSM), Model Robust Regression 2 (MRR2) is a good choice of a statistical model. MRR2 is a hybrid model obtained from the combination of both the classical parametric Ordinary Least Squares (OLS) and a nonparametric Local Linear Regression (LLR) via a mixing parameter. LLR portion of MRR2 utilizes kernel weights derived from the simplified product Gaussian function. A motivation for this paper is derived from the fact that, since the OLS residuals are the equivalence of the response that the LLR portion is designed to estimate, then the kernel weight at each data point should reflect the relative magnitude of the OLS residual at each data point. In order to improve on the performance of MRR2, we therefore propose a robustification of the kernel weights using two different linearly transformed residuals vectors from the OLS component. Data from real experiments, statistical literature as well as simulation study were analyzed. Comparison of results shows that the MRR2 that utilizes the proposed robustified kernel weights outperforms OLS, LLR and the MRR2 that utilizes existing kernel weights by considerably wide margins. For the minimization of surface roughness of a machined mild steel material (EN10) in particular, the optimal cutting speed, feed rate and depth of cut of 254.3979m/min, 0.1774mm/rev and 0.4388mm, respectively, obtained by MRR2 utilizing one of the proposed techniques for robustifying kernel weights gave a desirability of 99.4%. This implies that the optimal value of each of the four roughness parameters collectively meets 99.4% of the process requirements.

Keywords: Semi-parametric regression, response surface methodology, multiple response optimization, kernel weights, surface roughness.

INTRODUCTION

The focus of this paper is on the application of response surface methodology (RSM) in the determination of optimal setting of one or more explanatory variables (e.g. the cutting parameters) that optimizes a given response variable (e.g. surface roughness).

Response Surface Methodology (RSM) is a collection of mathematical techniques employed by statisticians and engineers in the modeling and analysis of problems in which a response of interest is influenced by one or more explanatory variables (Box and Wilson, 1951; Myers *et al*., 2009). The ultimate goal of RSM is to determine the value(s) of the explanatory variables that will optimize response(s) via a predictive model(s) fitted to the small sample data generated from a designed experiment (Wan and Birch, 2011).

In machining processes, metal cutting happens to be one of the most widely used manufacturing processes in engineering industries and factories (Reddy *et al*., 2011). Metal cutting operation plays an important role in reducing a particular work piece from the original stock to the desired shape and dimension with certain level of surface roughness (Reddy and Mallampati, 2012).

Surface roughness is a common criterion for evaluating the quality of a product. It affects factors such as friction, ease of lubricant, electrical and thermal conductivity, geometry tolerance, etc (Rodriques *et al*., 2012; Sharma *et al*., 2012; Makadia and Nanavati, 2013). Surface roughness has been found to influence properties such as

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wear resistance and fatigue strength and the functionality of the machined components (Pradeep *et al*., 2008).

Microscopic examination of machined metallic surfaces reveals projections which consist of crests (peaks) and troughs (valleys), of varying distances, heights and depths, resulting in irregularities, protuberances and ridges on the surface of the machined part. The differences and distances between these crests and troughs give a measure of the surface texture or more specifically roughness of the surface.

For the purpose of this study, four aspects of surface roughness of the mild steel material were measured and recorded in four parameters, namely, the arithmetic total average roughness (R_a), the average distance between the highest peak and lowest valley in each length of test piece(R_z),the root mean square average of the profile heights over the evaluation length (R_q), and the maximum height of the profile (R_t), all in microns (μm).

The interplay of different values of the cutting conditions such as cutting speed, feed rate and deep of cut results in different values of surface roughness of the work piece. Hence, in order to achieve the economic objective of a machining process, the optimal cutting conditions for a material of a given specification (including the physical and chemical properties) must be determined. To do this using RSM, predictive empirical model is established via data generated from a statistically designed experiment such as Central Composite Design (CCD) and Box-Behken Design. Subsequently, the empirical model so obtained is interfaced with an optimization program to get the setting cutting parameters that minimize the surface roughness for a particular material (Del Castillo, 2007).

Accurately determined optimal cutting conditions and surface roughness provides better opportunity for a significant improvement in the quality of manufactured components in terms of wear resistance, fatigue strength, ease of lubricant, etc, and an overall reduction in manufacturing costs as a result of reduction in material wastage, reduction in man-hour, power consumption, etc. Knowledge about optimal cutting conditions play a significant role in the efficient use of machines and machine tools and motivates machinists to handle them as efficiently as possible in order to maximize returns from them (Babu *et al*., 2011).

As earlier stated, before the optimization phase of RSM, the functional relationship between the response variable y and the k explanatory variables $x_1, x_2, \ldots, x_k$, must be established (Pickle *et al*., 2008).

A first step towards this goal is to assume that this relationship is in a mathematical form represented as:

$$y_i = f(x_{i1}, x_{i2}, \ldots, x_{ik}) + \varepsilon_i, \quad i = 1,2,\ldots,n \qquad (1)$$

where the mean function f denotes the true but unknown relationship between the response variable and the k explanatory variables, $y_i, i = 1,2,\ldots,n,$ is the value of the response at ith data point, $x_{ij},\ i = 1,2,\ldots,n, j = 1,2,\ldots,k,$ denotes the value of the j^{th} explanatory variable at the i^{th} data point, $\varepsilon_i,\ i = 1,2,\ldots,n,$ is the error term at the i^{th} data point, where $\boldsymbol{\varepsilon} \sim N(0,\sigma^2), and\ n$ is the sample size (Wan and Birch, 2011).

Next is to choose a regression model to use for estimating the functional form of f in (1). Regression methods applied in RSM include OLS, LLR and MRR2 (Montgomery 2009; He *et al*., 2012).

OLS model comes handy in scenarios where the researcher has perfect knowledge of a polynomial that adequately approximates f in (1) with a very high degree of accuracy (Pickle *et al*., 2008; Shah *et al*., 2004).

The OLS estimate,$\hat{y}_i^{(OLS)}$, of the response in the i^{th} data point is given as:

$$\hat{y}_i^{(OLS)} = \boldsymbol{x_i}(\boldsymbol{X^T X})^{-1}\boldsymbol{X^T y}, \qquad (2)$$

where $\boldsymbol{y}$ is a $n \times 1$ vector of response, **X** is a n × p model matrix, p is the number of model parameters (coefficients) in the assumed model, $\boldsymbol{X^T}$is the transpose of the matrix **X,** and $\boldsymbol{x_i}$ is the i^{th} row vector of the matrix $\boldsymbol{X}$ (Pickle *et al*., 2008; Edionwe and Mbegbu, 2014).

In matrix notation, the vector of OLS estimated response is expressed as:

$$\hat{\boldsymbol{y}}^{(OLS)} = \begin{bmatrix} \boldsymbol{h}_1^{(OLS)} \\ \boldsymbol{h}_2^{(OLS)} \\ \vdots \\ \boldsymbol{h}_n^{(OLS)} \end{bmatrix} \boldsymbol{y} = \boldsymbol{H}^{(OLS)}\boldsymbol{y}, \qquad (3)$$

where the vector $\boldsymbol{h}_i^{(OLS)} = \boldsymbol{x_i}(\boldsymbol{X^T X})^{-1}\boldsymbol{X^T}$is the i^{th} row of the $n \times n$ OLS Hat matrix $\boldsymbol{H}^{(OLS)}$.

OLS is robust to the polynomial specified to approximate f in (1) but requires several assumptions to be met for valid interpretation of its parameter estimates. Furthermore, it performs poorly if the assumed polynomial model is inadequate for the data at hand (Wan and Birch, 2011).

Mathematically, LLR estimate, $\hat{y}_i^{(LLR)}$of y_i, is given as:

$$\hat{y}_i^{(LLR)} = \tilde{\boldsymbol{x}}_i(\tilde{\boldsymbol{X}}^T\boldsymbol{W}_i\tilde{\boldsymbol{X}})^{-1}\tilde{\boldsymbol{X}}^T\boldsymbol{W}_i\boldsymbol{y} = \boldsymbol{h}_i^{(LLR)}\boldsymbol{y} \qquad (4)$$

where $\tilde{\boldsymbol{x}}_i$is the ith row of the LLR model matrix $\tilde{\boldsymbol{X}}$given as:

$$\widetilde{\boldsymbol{X}} = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{bmatrix}, \quad (5)$$

$\boldsymbol{W_i}$ is an $n \times n$ diagonal matrix of the weights for estimating the ith response. The weights utilized in the LLR model are derived from one of the several kernel functions such as the Gaussian kernel function (Fan and Gibjels, 1992; Anderson-Cook and Prewitt, 2005; Zheng *et al.*, 2013).

The rth-entry, say w_r of kernel weights matrix, $\boldsymbol{W_i}$ for estimating $y_i$in (4) is obtained from the product kernel as:

$$w_r = \prod_{j=1}^{k} K\left(\frac{x_{ij}-x_{rj}}{b}\right) / \sum_{i=1}^{n} \prod_{j=1}^{k} K\left(\frac{x_{ij}-x_{rj}}{b}\right), i = 1,2,\ldots,n, j = 1,2,\ldots,k, r = 1,2,\ldots,n, \quad (6)$$

where $K\left(\frac{x_{ij}-x_{rj}}{b}\right) = e^{-\left(\frac{x_{ij}-x_{rj}}{b}\right)^2}$ is the simplified Gaussian function. For response surface data, the kernel weights (designed to lie in[0,1]), are derived via the simplified Gaussian function of smoothed differences between the value of the explanatory variable at each data point and a specified data point of interest, and b, $0 < b \leq 1$, is the smoothing parameter (bandwidth) (Mays *et al.*, 2001; Edionwe *et al.*, 2016).

Hence, the $n \times n$ diagonal matrix kernel weights $\boldsymbol{W_i}, i = 1,2,\ldots,n,$ can be express as:

$$\boldsymbol{W_i} = \begin{bmatrix} w_1 & 0 & \cdots & 0 \\ 0 & w_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & w_n \end{bmatrix},$$

$$= \begin{bmatrix} \left(\frac{\prod_{j=1}^{k} e^{-\left(\frac{x_{ij}-x_{1j}}{b}\right)^2}}{\sum_{i=1}^{n}\prod_{j=1}^{k} e^{-\left(\frac{x_{ij}-x_{1j}}{b}\right)^2}}\right) & 0 & \cdots & 0 \\ 0 & \left(\frac{\prod_{j=1}^{k} e^{-\left(\frac{x_{ij}-x_{2j}}{b}\right)^2}}{\sum_{i=1}^{n}\prod_{j=1}^{k} e^{-\left(\frac{x_{ij}-x_{2j}}{b}\right)^2}}\right) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \left(\frac{\prod_{j=1}^{k} e^{-\left(\frac{x_{ij}-x_{nj}}{b}\right)^2}}{\sum_{i=1}^{n}\prod_{j=1}^{k} e^{-\left(\frac{x_{ij}-x_{nj}}{b}\right)^2}}\right) \end{bmatrix}$$

In matrix notation, the LLR estimates of the response can be expressed as:

$$\widehat{\boldsymbol{y}}^{(LLR)} = \boldsymbol{H}^{(LLR)}\boldsymbol{y} = \begin{bmatrix} \boldsymbol{h}_1^{(LLR)} \\ \boldsymbol{h}_2^{(LLR)} \\ \vdots \\ \boldsymbol{h}_n^{(LLR)} \end{bmatrix} \boldsymbol{y}, \quad (7)$$

where $\boldsymbol{H}^{(LLR)}$ is the $n \times n$ LLR Hat matrix, and $\boldsymbol{h}_i^{(LLR)} = \widetilde{\boldsymbol{x}}_i(\widetilde{\boldsymbol{X}}^T\boldsymbol{W_i}\widetilde{\boldsymbol{X}})^{-1}\widetilde{\boldsymbol{X}}^T\boldsymbol{W_i}$ is the i^{th} row vector of the LLR Hat matrix for estimating y_i.

For small sample studies such as RSM, the optimal bandwidth is chosen based on the minimization of the Penalized Prediction Error Sum of Squares ($PRESS^{**}$) criterion given as:

$$PRESS^{**}(b) = \frac{PRESS}{n - trace(H^{(LLR)}(b)) + (n-k-1)\frac{SSE_{max}-SSE_b}{SSE_{max}}}, \quad (8)$$

where$PRESS = \sum_{i=1}^{n}\left(y_i - \hat{y}_{i,-i}^{(LLR)}\right)^2$,$\hat{y}_{i,-i}^{(LLR)}$ is the leave-one-out cross-validation estimate of y_i with the ith observation left out, SSE_{max} is the maximum Sum of Squared Errors obtained as b tends to infinity, SSE_b is the Sum of Squared Errors associated with a particular value of b, $tr\left(H^{(LLR)}b\right)$ is the trace of the LLR Hat matrix (Mays *et al.*, 2000; Pickle *et al.*, 2008).

LLR is flexible and sensitive to interpretation of local trends in a data particular when applied to fit data which consist of a single explanatory variable and a fairly large sample size. LLR is generally found to perform poorly for small sample data generated from two or more explanatory variables (Pickle *et al.*, 2008; Wan and Birch, 2011; Geenens, 2011; Edionwe *et al.*, 2018).

MRR2 model combines both OLS estimates and a portion of the LLR estimates of OLS residuals via a mixing parameter, λ, where $0 \leq \lambda \leq 1$ (Mays *et al.*, 2001; Mays and Birch, 2002; Edionwe *et al.*, 2018).

Mathematically, MRR2 estimate, $\hat{y}_i^{(MRR2)}$of y_i, is given as:

$$\hat{y}_i^{(MRR2)} = \boldsymbol{x_i}(\boldsymbol{X^T X})^{-1}\boldsymbol{X^T y} + \lambda\tilde{\mathbf{x}}_{\mathbf{i}}(\widetilde{\boldsymbol{X}}^T\boldsymbol{W_i}\widetilde{\boldsymbol{X}}^T)^{-1}\widetilde{\boldsymbol{X}}^T\boldsymbol{W_i}(\boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X^T X})^{-1}\boldsymbol{X^T})\boldsymbol{y}, \quad (9)$$

$$= \boldsymbol{x_i}(\boldsymbol{X^T X})^{-1}\boldsymbol{X^T y} + \lambda\tilde{\mathbf{x}}_{\mathbf{i}}\left(\widetilde{\boldsymbol{X}}^T\boldsymbol{W_i}\widetilde{\boldsymbol{X}}\right)^{-1}\widetilde{\boldsymbol{X}}^T\boldsymbol{W_i}\boldsymbol{r}^{(OLS)}, \quad (10)$$

where $\boldsymbol{r}^{(OLS)}$ is the $n \times 1$ vector of the OLS residuals, $\boldsymbol{I}$ is an $n \times n$ identity matrix and $\boldsymbol{W_i}$ is a $n \times n$ diagonal weights matrix for estimating the i^{th} entry of $\boldsymbol{r}^{(OLS)}$ and derived as in (4) for LLR using (8) for its bandwidth with $\hat{y}_{i,-i}^{(LLR)}$ and $H^{(LLR)}$ replaced with $\hat{y}_{i,-i}^{(MRR2)}$ and H^{MRR2}, respectively, and every other term retaining the previous definition in (8).

Equation (10) may be expressed in matrix form as:

$$\boldsymbol{y}^{(MRR2)} = \begin{bmatrix} \boldsymbol{h}_1^{(MRR2)} \\ \boldsymbol{h}_2^{(MRR2)} \\ \vdots \\ \boldsymbol{h}_n^{(MRR2)} \end{bmatrix} \boldsymbol{y} = \boldsymbol{H}^{(MRR2)}\boldsymbol{y}, \quad (11)$$

where $\boldsymbol{h}_i^{(MRR2)} = \boldsymbol{x}_i(\boldsymbol{X^T X})^{-1}\boldsymbol{X^T} + \lambda \tilde{\mathbf{x}}_i(\tilde{\boldsymbol{X}}^T \boldsymbol{W}_i \tilde{\boldsymbol{X}}^T)^{-1}\tilde{\boldsymbol{X}}^T \boldsymbol{W}_i(\boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X^T X})^{-1}\boldsymbol{X^T})$ is the i^{th} row of the $n \times n$ MRR2 Hat matrix, $\boldsymbol{H}^{(MRR2)}$ (Pickle *et al*., 2008).

The optimal value of λ, is selected based on the minimization of a form of the $PRESS^{**}$ criterion given as:

$$PRESS^{**}(\lambda) = \frac{PRESS}{n - tr\left(H^{(MRR2)}(b^*,\lambda)\right) + (n-k-1)\frac{SSE_{max} - SSE_{(b^*,\lambda)}}{SSE_{max}}}, \quad (12)$$

where $PRESS = \sum_{i=1}^{n}\left(y_i - \hat{y}_{i,-i}^{(MRR2)}(b^*,\lambda)\right)^2$, $\hat{y}_{i,-i}^{(MRR2)}(b^*,\lambda)$ is the leave-one-out cross-validation MRR2 estimate of y_i given the optimal bandwidth b^* and a candidate value of the mixing parameter λ, $SSE_{b^*\lambda}$ is the Sum of Squared of Errors given the optimal bandwidth and a candidate value of λ, $tr\left(H^{(MRR2)}(b^*,\lambda)\right)$ is the trace of MRR2 hat matrix given the optimal bandwidth and a candidate value of λ (Mays and Birch, 2002; Wan and Birch, 2011).

The MRR2 inherits the advantages of OLS (Robustness) and LLR (flexibility) while trying to minimize the limitations from both methods **(**Mays *et al*., 2001**)**. LLR forms part of MRR2. Therefore, it follows that the factors that put a limitation on the performance of LLR also more or less put a limitation on the performance of MRR2.

This paper focuses on improving the performance of MRR2 via the robustification of the kernel weights utilized by the LLR component.

Once the data has been modelled RSM procedure proceeds to the optimization phase where the resulting fitted curve (from the estimated responses from equations (2), (4), (9) is used for determining the setting of the explanatory variables that optimizes the response based on the production requirement (Johnson and Montgomery, 2009; Mondal and Datta, 2011).

When dealing with studies that involve m response, $m > 1$, it is essential that we get an optimal setting of the explanatory variables that simultaneously optimize all the responses with respect to their individual production requirements. For this purpose, the desirability function comes handy. The most popular criterion applied in the optimization of multiple responses is the Desirability function (Harrington, 1965; Derringer and Suich, 1980; Adalarasan and Santhanakumar, 2015).

The overall objective of the Desirability criterion is to obtain the setting of the explanatory variables that maximizes the geometric mean (D) of all the individual desirability measures, where D is given as:

$$D = maximize\left(\left(\prod_{p=1}^{m} d_p\left(\hat{y}_p(\boldsymbol{x})\right)\right)^{1/m}\right), \quad (13)$$

where $d_p\left(\hat{y}_p(\boldsymbol{x})\right)$, $0 \le d_p\left(\hat{y}_p(\boldsymbol{x})\right) \le 1, p = 1,2,\ldots,m,$

are the scalar measures obtained from the transformation of the estimated response in equations (2), (4), (9) with respect to the production requirement of a response (Wu and Hamada, 2000; Wan and Birch, 2011).

If the response is of nominal-the-better (NTB) type where the p^{th} response acceptable value lies between an upper limit, U and a lower limit, L, $d_p\left(\hat{y}_p(\boldsymbol{x})\right)$ is given as:

$$d_p\left(\hat{y}_p(\boldsymbol{x})\right) = \begin{cases} 0 & \hat{y}_p(\boldsymbol{x}) < L \\ \left\{\frac{\hat{y}_p(x) - L}{\emptyset - L}\right\} & L \le \hat{y}_p(\boldsymbol{x}) < \emptyset, \\ \left\{\frac{U - \hat{y}_p(x)}{U - \emptyset}\right\} & \emptyset \le \hat{y}_p(\boldsymbol{x}) \le U, \\ 0 & \hat{y}_p(\boldsymbol{x}) > U, \end{cases} \quad (14)$$

where $\emptyset$ is the target value of the p^{th} response.
If the objective is to maximize the p^{th} response, $d_p\left(\hat{y}_p(\boldsymbol{x})\right)$ is given as:

$$d_p\left(\hat{y}_p(\boldsymbol{x})\right) = \begin{cases} 0 & \hat{y}_p(\boldsymbol{x}) < L, \\ \left\{\frac{\hat{y}_p(x) - L}{\emptyset - L}\right\} & L \le \hat{y}_p(\boldsymbol{x}) \le \emptyset, \\ 1 & \hat{y}_p(\boldsymbol{x}) > \emptyset, \end{cases} \quad (15)$$

where $\emptyset$ is interpreted as large enough value of the p^{th} response.
If the objective is to minimize the p^{th} response, $d_p\left(\hat{y}_p(\boldsymbol{x})\right)$ is given as:

$$d_p\left(\hat{y}_p(\boldsymbol{x})\right) = \begin{cases} 1 & \hat{y}_p(\boldsymbol{x}) < \emptyset, \\ \left\{\frac{U - \hat{y}_p(x)}{U - \emptyset}\right\} & \emptyset \le \hat{y}_p(\boldsymbol{x}) \le U, \\ 0 & \hat{y}_p(\boldsymbol{x}) > U, \end{cases} \quad (16)$$

where $\emptyset$ is a small enough value of the p^{th} response.
In this paper, all the optimization routines regarding the minimization of the PRESS** criterion for the choice of optimal bandwidth, optimal mixing parameter and the

maximization of the desirability function for the determination of the optimal value of the response and the corresponding setting of the explanatory variables in equations (8), (12) and (13), respectively, were carried out using the Genetic Algorithm (GA) optimization toolbox available in Matlab software.

GA is based on natural selection and other genetic concepts including population, chromosomes, selection, crossover, mutation, etc. (Heredia-Langner *et al.*, 2004; Chen and Ye, 2009). GA can be applied to solve several difficult optimization problems including the one in which the objective function lacks closed form expression as in the case with LLR and MRR2 (Alvarez *et. al.,* 2009; Thongsook *et al.,* 2014; Yeniay, 2014).

MATERIALS AND METHODS

Experimental Procedure

Material: Mild Steel (EN10) rods of diameter 25mm with Carbon Equivalent (CE) of 0.34 were purchased from local source. Mild steel (EN10) materials find wide range of industrial applications including the manufacture of rifle barrels and gear wheel housings. The mechanical and chemical properties, certified according to ISO 9001 Standard at 20^o Centigrade, are presented in Table 1 and Table 2, respectively.

Table 1. Chemical composition of mild steel (EN10)

S No.	Elements	Composition
1	Carbon (C)	0.170
2	Silicon (Si)	0.310
3	Manganese (Mn)	0.780
4	Phosphorus (P)	0.030
5	Sulfur (S)	0.030
6	Titanium (Ti)	0.000
7	Copper (Cu)	0.310
8	Nickel (Ni)	0.070
9	Chromium (Cr)	0.060
10	Molybdenum (Mo)	0.008
11	Vanadium (V)	0.000
12	Aluminium (Al)	0.027

Table 2. Mechanical properties of mild steel (EN10).

S No.	Properties	Value
1	Yield Strength (Re)	280.40N/m^2
2	Tensile Strength (Rm)	408.30N/m^2
3	Rm /Re Ratio	1.46
4	Elongation	27.40%

In order to remove contaminants from the mild steel rod, a 1mm diameter was first rough turned to get a 24mm working diameter.

The rods were then cut into test pieces of length 60mm. Machining was carried out using a CNC lathe machine (GT-1628 CNC) manufactured by Ganesh Machinery. The CNC Lathe machine is shown in Figure 1.

Fig. 1. A CNC lathe machine.

The experiment was based on a Central Composite Design (CCD) in $k = 3$ machining parameters represented by the explanatory variables, namely cutting speed in rev/min (x_1), feed rate in mm/rev (x_2) and depth of cut in mm (x_3). The decision to use a CCD was informed by the need to capture, as much as possible, the quadratic effects of each machining parameters. Moreover, each level of the experiment was replicated three times in order to reduce the effect of random errors (Del Castillo, 2007; Myers *et al.*, 2009).

The ranges of values for the machining parameters were based on standard charts in machining technology (Jain, 2009). The three portions of a CCD comprising five standard distinct values of each of the k explanatory variables used in the paper are as follows:

(i) The $2^k = 8$ Corner points (Values) with $x_j =$ Low Value and High Value for $j = 1,2,\dots,k$ which form the factorial portion of the design,

(ii) $k = 3$ Centre points with $x_j = (\text{High Value} + \text{Low Value})/2$, for $j = 1,\dots,k$;

(iii) $2k = 6$ Star or Axial points with $x_j = -\alpha =$ High Value $-2.6818\times$(High Value – Centre Value), and $x_j = \alpha =$ High Value $+ 0.6818 \times (\text{High Value} - \text{Centre Value})$.

The summary of the five standard values obtained for each of the machining parameters, the resulting CCD with the i =17 data points (from i, ii and iii above) and the corresponding responses from three replicates are presented in Table 3, 4 and 5, respectively. Also included in Table 3 are the coded values, $x_{ij}, i = 1,2,\dots,n, j = 1,2\dots,k,$ of each of the machining parameters, where $0 \le x_{ij} \le 1$, for the purpose of the LLR model (Pickle et al., 2008; Wan and Birch, 2011). To do this, we use the formula:

$$\boldsymbol{x}_{(coded\ value)ij} = \frac{x_{(real\ value)ij} - \min(x_{real\ value})}{\max(x_{real\ value}) - \min(x_{real\ value})},$$

where $\boldsymbol{x}_{coded\ value}$ is the coded value of $\boldsymbol{x}_{ij}$, $\mathbf{min}(\boldsymbol{x}_{real\ value})$ and $\max(\boldsymbol{x}_{real\ value})$ are the minimum and maximum values of each explanatory variable, respectively.

Figure 2 shows the rods (test pieces) after being machined with a coated cemented carbide tool according to each setting of the machining parameters in Table 4. After machining, the test pieces were allowed to cool to room temperature. The tester (TR200) used to measure the roughness parameters (R_a, R_z, R_q, R_t) of each of the test pieces is shown in Figure 3.

Table 3. The five values of each of the machining parameters used to generate the CCD.

Parameter	**-α**	**Low Value**	**Medium**	**High value**	**+α**
Cutting Speed	215.91	250	300	350	384.09
Feed Rate	0.0159	0.05	0.10	0.15	0.1841
Depth of Cut	0.0111	0.11	0.2550	0.40	0.4989

Table 4. The CCD showing the real experimental values and the coded values.

i	x_1	x_2	x_3	**Coded Values**		
				x_1	x_2	x_3
1	250.00	0.0500	0.1100	0.2030	0.2030	0.2030
2	250.00	0.0500	0.4000	0.2030	0.2030	0.7970
3	250.00	0.1500	0.1100	0.2030	0.7970	0.2030
4	250.00	0.1500	0.4000	0.2030	0.7970	0.7970
5	350.00	0.0500	0.1100	0.7970	0.2030	0.2030
6	350.00	0.0500	0.4000	0.7970	0.2030	0.7970
7	350.00	0.1500	0.1100	0.7970	0.7970	0.2030
8	350.00	0.1500	0.4000	0.7970	0.7970	0.7970
9	215.91	0.1000	0.2550	0.0000	0.5000	0.5000
10	384.09	0.1000	0.2550	1.0000	0.5000	0.5000
11	300.00	0.0159	0.2550	0.5000	0.0000	0.5000
12	300.00	0.1841	0.2550	0.5000	1.0000	0.5000
13	300.00	0.1000	0.0111	0.5000	0.5000	0.0000
14	300.00	0.1000	0.4989	0.5000	0.5000	1.0000
15	300.00	0.1000	0.2550	0.5000	0.5000	0.5000
16	300.00	0.1000	0.2550	0.5000	0.5000	0.5000
17	300.00	0.1000	0.2550	0.5000	0.5000	0.5000

Table 5. The three replicates of surface roughness in the four parameters and their averages using the CCD in Table 4.

i	Surface Roughness															
	R_a Rep1	R_a Rep2	R_a Rep3	R_a Ave	R_z Rep1	R_z Rep2	R_z Rep3	R_z Ave	R_q Rep1	R_q Rep2	R_q Rep3	R_q Ave	R_t Rep1	R_t Rep2	R_t Rep3	R_t Ave
1	1.5040	1.4855	1.5123	**1.5006**	4.3319	4.9666	4.1706	**4.4897**	1.8069	1.7888	2.1628	**1.9195**	3.6991	4.4401	5.2156	**4.4516**
2	0.7409	0.8012	0.6923	**0.7448**	2.4990	2.1043	3.3800	**2.6611**	1.0218	1.0091	0.2332	**0.7547**	1.6702	2.0016	1.0328	**1.5682**
3	0.7699	0.6989	0.8442	**0.7710**	2.6040	3.0542	1.9636	**2.5406**	0.9660	1.0112	0.5437	**0.8403**	2.0039	1.5404	1.2299	**1.5914**
4	2.6808	3.0019	2.8760	**2.8529**	2.8019	3.3439	4.6896	**3.6118**	3.7171	2.9696	3.7794	**3.4887**	5.9806	6.1720	7.2718	**6.4748**
5	1.2520	1.4019	1.2701	**1.3080**	3.9793	3.7995	4.3166	**4.0318**	1.8840	1.7701	1.4939	**1.7160**	3.1377	2.7614	5.1304	**3.6765**
6	0.9905	0.9109	1.0887	**0.9967**	2.6994	3.8377	3.8546	**3.4639**	1.4533	0.9998	1.1802	**1.2111**	1.9299	2.8116	2.8368	**2.5261**
7	0.5101	0.7004	0.2928	**0.5011**	1.8533	2.0200	1.6950	**1.8561**	0.5309	1.0041	0.2956	**0.6102**	1.7007	1.3919	0.8065	**1.2997**
8	1.7106	1.3010	1.3813	**1.4643**	1.8620	2.0989	1.5591	**1.8400**	2.0102	1.6569	2.0029	**1.8900**	4.1040	4.6032	2.9142	**3.8738**
9	0.8303	0.6104	0.9530	**0.7979**	3.8281	3.7799	5.3382	**4.3154**	0.7993	0.7886	1.4007	**0.9962**	2.2510	1.9778	2.4939	**2.2409**
10	0.3063	0.4430	0.3700	**0.3731**	2.5072	2.1713	1.6755	**2.1180**	0.4001	0.7073	0.0167	**0.3747**	0.7515	1.0980	0.5742	**0.8079**
11	1.2913	1.4254	1.3369	**1.3512**	3.9544	2.9797	4.7974	**3.9105**	1.7608	1.3387	2.0788	**1.7261**	3.9486	3.1774	4.4021	**3.8427**
12	1.7778	2.0080	1.1909	**1.6589**	2.8330	2.5457	1.7475	**2.3754**	2.3050	2.5158	1.2704	**2.0304**	5.0009	5.3001	3.9292	**4.7434**
13	1.1870	0.9853	1.0497	**1.0740**	3.5935	2.9007	2.6174	**3.0372**	1.6100	1.2787	0.7818	**1.2235**	3.2116	2.5994	2.6898	**2.8336**
14	2.5885	2.4007	3.0130	**2.6674**	2.8077	3.9087	4.7658	**3.8274**	3.4936	2.6455	4.5700	**3.5697**	6.3882	7.1597	9.4939	**7.6806**
15	1.5241	1.7602	1.2265	**1.5036**	5.1100	4.9070	3.5367	**4.5179**	2.2075	1.9889	2.6970	**2.2978**	4.9082	4.6619	5.2880	**4.9527**
16	1.4674	1.3399	1.7326	**1.5133**	4.2890	4.6920	4.2916	**4.4242**	1.9393	1.9592	2.0316	**1.9767**	5.1076	4.7783	5.1954	**5.0271**
17	1.7118	1.4161	1.3694	**1.4991**	4.1166	4.4581	4.3541	**4.3096**	1.9068	2.0091	2.6754	**2.1971**	4.7678	4.8477	5.9644	**5.1933**

The test pieces were allowed to cool to room temperature and their surface roughness values in the four parameters (R_a, R_z, R_q and R_t) obtained using the surface roughness tester shown in Figure 3.

Fig. 2. Sample of the mild steel (EN10) test pieces.

Fig. 3. Surface roughness tester (TR200).

Methodology

The estimate from the OLS component of MRR2 is fixed for a given polynomial specified for $f$in equation (1). Hence, researchers usually look to LLR component since improvement in the LLR component seamlessly carries over to MRR2 model.

A motivation for this research is that OLS residuals reflect the inadequacy of the OLS component of MRR2, thus kernel weights derived with embellishments from the OLS residuals would provide superior kernel weights for correcting the disparity in OLS estimates and the true response, and consequently, better estimates than those obtained solely from the explanatory variables.

The kernel weights in equation (6) can be represented in a general form as:

$$w_r = q_r \left[\prod_{j=1}^{k} K\left(\frac{x_{ij}-x_{rj}}{b}\right) / \sum_{i=1}^{n} \prod_{j=1}^{k} K\left(\frac{x_{ij}-x_{rj}}{b}\right)\right], i = 1,2,\dots,n, j = 1,2,\dots,k,\ r = 1,2,\dots,n, \quad (17)$$

where $K\left(\frac{x_{ij}-x_{rj}}{b}\right) = e^{-\left(\frac{x_{ij}-x_{rj}}{b}\right)^2}$ is the simplified Gaussian function, $q_i = 1, i = 1,2,\dots,n.$

The kernel weights w_i, $i = 1,2,\dots,n$, are derived from the simplified Gaussian function of the smoothed differences between the value of the explanatory variable at each data point and a specified point of interest, thus lacking reference to the OLS residuals the LLR component is designed to estimate. Therefore, the weights derived lack consonance with the relative sizes and shape of the OLS residuals that the LLR component is designed to estimate.

To correct this defect in the existing kernel weights, we propose a substitution of q_r in equation (17) with kernel weights derived from the simple Gaussian function of the residuals from the OLS component of MRR2 given as $q_r = e^{-\left(\frac{p_i-p_r}{b_1}\right)^2}$,where p_i, $i = 1,2,\dots,n$, is the transformed OLS residuals, $y_i - \hat{y}_i^{(OLS)}$, p_r is the r^{th} transformed OLS residuals, b_1 is the bandwidth for smoothing the differences between the transformed OLS residuals at i^{th} and the r^{th} data points. This is to ensure that the kernel weights derived reflect the relative the sizes of the OLS residuals at each of the data points.

The philosophy applied in the transformation is that a data point where the OLS residual,$e_i = y_i - \hat{y}_i^{(OLS)}$ is negative indicates that the OLS estimate is superfluous (i.e. larger than the true responsey_i) and therefore should be assigned a relatively smaller kernel weight than the one assigned to a data point where $e_i = y_i - \hat{y}_i^{(OLS)}$ is either zero or positive.

We now present two distinct procedures for utilizing the OLS residuals in the determination of p_i, $i = 1,2,\dots,n.$

(i) p_i from Linearly Transformed Ranks of OLS Residuals

Let $\boldsymbol{\theta}$ denotes the $n \times 1$ vector of the residuals from the OLS component of MRR2. First, we obtain the ranks, θ_i,$i = 1,2,\dots,n$,of the values of OLS residuals, $e_i = y_i - \hat{y}_i^{(OLS)}$, $i = 1,2,\dots n$, in an ascending order, that is $\text{Rank}(e_i) = \theta_i$, where $\theta_{i+1} > \theta_i$, for every $i = 1,2,\dots,n$. This ensures that relatively larger ranks (weights) are assigned to relatively larger OLS residuals.

Next, we obtain the linearly transformed ranks of the OLS residuals, denoted as p_i, $i = 1,2,\dots,n$, to ensure they lie in the interval $[0, 1]$ in consonance with standard values of kernel weights and that of the explanatory variables:

$$p_i = \frac{\theta_i - \min(\boldsymbol{\theta})}{\max(\boldsymbol{\theta}) - \min(\boldsymbol{\theta})} = \frac{\theta_i - 1}{n-1}, i = 1,2,\dots,n, \quad (18)$$

(ii) p_i from Linearly Translated and Transformed OLS Residuals

First, we obtain the horizontal translation of the OLS residuals, $\boldsymbol{e_i} = y_i - \hat{y}_i^{(OLS)}$, in order to assign a number t_i to each of the OLS residuals, where

$$t_i = e_i + |\min(e_i)| + 1, i = 1,2,\dots,n, \quad (19)$$

Again, (19) ensures that relatively larger numbers (weights) are assigned to relatively larger OLS residuals.

Next, we get the linearly translated OLS residuals t_i to ensure they lie in the interval $[0, 1]$.

$$p_i = \frac{t_i - \min(\boldsymbol{t})}{\max(\boldsymbol{t}) - \min(\boldsymbol{t})} = \frac{t_i - 1}{\max(\boldsymbol{t}) - 1},\ i = 1,2,\dots,n, \quad (20)$$

where $\boldsymbol{t}$ is the vector of the linearly translated OLS residuals.

Therefore, the product Gaussian kernel weight $w_r$of $\boldsymbol{W_i}$ in (17), $i = 1,2,\dots,n$, for MRR2 estimate $\hat{y}_i^{MRR2}$ of y_i is obtained as:

$$w_r = e^{-\left(\frac{p_i-p_r}{b_1}\right)^2} \left[\prod_{j=1}^{k} K\left(\frac{x_{ij}-x_{rj}}{b_2}\right) / \sum_{i=1}^{n} \prod_{j=1}^{k} K\left(\frac{x_{ij}-x_{rj}}{b_2}\right)\right], \quad (21)$$

where $i = 1,2,\dots,n, j = 1,2,\dots,k,\ r = 1,2,\dots,n,$ and p_i is any of the transformed OLS residuals in (18) or (20).

RESULTS AND DISCUSSION

Surface Rough Data

The data for analysis is extracted from Table 4 (the coded values of each explanatory variable) and Table 5 (the average of the three replicates of each of the roughness parameters) and presented in Table 6. For each roughness parameters, a full quadratic regression model was found to give the highest $R^2$and a p-value of less than 0.05 for each term.

The process requirements considered in the study are as follows:

Minimize $R_a$with target value $\emptyset$ =0.01 and upper bound U=1.2911, where U= mean of Ra;

Minimize R_z with target value $\emptyset$ =0.01 and upper bound U =3.2643, where U=mean of Rz;
Minimize R_q with target value $\emptyset$ =0.01 and upper bound U=1.6176, where U=mean of Rq;
Minimize R_t with target value $\emptyset$ =0.01, and upper bound U =3.6499, where U=mean of Rt.
The goodness of fit and the optimization results are presented in Table 7 and Table 8, respectively. Figure 4 shows the graphs of the residuals of each regression model as it applies to each of the four responses (that is the four parameters of surface roughness).

For ease of comparison, the results from MRR2 that utilizes the transformed OLS residuals in equation (18) and equation (20) are designated $MRR2_{rank}$and $MRR2_{+ve}$, respectively. The best values for each statistics across the responses are in bold.

Table 6. Surface roughness data.

$\boldsymbol{i}$	Coded Values			Surface Roughness			
	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{x_3}$	$\boldsymbol{R_a}$	$\boldsymbol{R_z}$	$\boldsymbol{R_q}$	$\boldsymbol{R_t}$
1	0.2030	0.2030	0.2030	1.5006	4.4897	1.9195	4.4516
2	0.2030	0.2030	0.7970	0.7448	2.6611	0.7547	1.5682
3	0.2030	0.7970	0.2030	0.7710	2.5406	0.8403	1.5914
4	0.2030	0.7970	0.7970	2.8529	3.6118	3.4887	6.4748
5	0.7970	0.2030	0.2030	1.3080	4.0318	1.7160	3.6765
6	0.7970	0.2030	0.7970	0.9967	3.4639	1.2111	2.5261
7	0.7970	0.7970	0.2030	0.5011	1.8561	0.6102	1.2997
8	0.7970	0.7970	0.7970	1.4643	1.8400	1.8900	3.8738
9	0.0000	0.5000	0.5000	0.7979	4.3154	0.9962	2.2409
10	1.0000	0.5000	0.5000	0.3731	2.1180	0.3747	0.8079
11	0.5000	0.0000	0.5000	1.3512	3.9105	1.7261	3.8427
12	0.5000	1.0000	0.5000	1.6589	2.3754	2.0304	4.7434
13	0.5000	0.5000	0.0000	1.0740	3.0372	1.2235	2.8336
14	0.5000	0.5000	1.0000	2.6674	3.8274	3.5697	7.6806
15	0.5000	0.5000	0.5000	1.5036	4.5179	2.2978	4.9527
16	0.5000	0.5000	0.5000	1.5133	4.4242	1.9767	5.0271
17	0.5000	0.5000	0.5000	1.4991	4.3096	2.1971	5.1933

The OLS fitted models for each of four roughness parameters are as follows:

$\hat{\boldsymbol{R}}_a$=$1.0026 + 4.9097\boldsymbol{x_1} - 1.2062\boldsymbol{x_2} - 2.6438\boldsymbol{x_3} - 3.7850\boldsymbol{x_1^2} - 0.1068\boldsymbol{x_2^2} + 1.3558\boldsymbol{x_3^2} - 2.4343\boldsymbol{x_1x_2} - 0.9554\boldsymbol{x_1x_3} + 5.8274\boldsymbol{x_2x_3}$,

$\hat{\boldsymbol{R}}_z$= $2.6311 + 5.5699\boldsymbol{x_1} + 3.1550\boldsymbol{x_2} + 1.7057\boldsymbol{x_3} - 5.1394\boldsymbol{x_1^2} - 5.4344\boldsymbol{x_2^2} - 4.2770\boldsymbol{x_3^2} - 3.9695\boldsymbol{x_1x_2} + 0.2457\boldsymbol{x_1x_3} + 4.8912\boldsymbol{x_2x_3}$,

$\hat{\boldsymbol{R}}_q$=$0.8571 + 7.3939\boldsymbol{x_1} - 0.7712\boldsymbol{x_2} - 2.7168\boldsymbol{x_3} - 6.0627\boldsymbol{x_1^2} - 1.2915\boldsymbol{x_2^2} + 0.7819\boldsymbol{x_3^2} - 2.9500\boldsymbol{x_1x_2} - 1.0043\boldsymbol{x_1x_3} + 7.9327\boldsymbol{x_1x_2}$,

$\hat{\boldsymbol{R}}_t$=$1.8498 + 16.5329\boldsymbol{x_1} - 1.2062\boldsymbol{x_2} - 4.6038\boldsymbol{x_3} - 15.2073\boldsymbol{x_1^2} - 4.1327\boldsymbol{x_2^2} - 0.2765\boldsymbol{x_3^2} - 4.3583\boldsymbol{x_1x_2} - 0.8167\boldsymbol{x_1x_3} + 16.2842\boldsymbol{x_2x_3}$,

Table 7. Goodness of fit of the regression models for the surface roughness data.

response	Model	b_1/b_2	λ	DF	SSE	MSE	R^2	R^2_{Adj}	$PRESS^{**}$
R_a	OLS	-	-	7.0000	0.5036	0.0719	92.8803	83.7264	0.6148
	LLR	0.4372	-	6.3646	0.4699	0.0738	93.3567	83.2995	5.9631
	MRR2	0.3398	1.0000	3.2142	0.1283	0.0399	98.1860	90.9700	1.1044
	$MRR2_{rank}$	0.2157/0.2035	1.0000	1.8920	**0.0001**	**0.0000**	**99.9988**	**99.9901**	**0.3979**
	$MRR2_{+ve}$	0.1836/0.2210	1.0000	2.0052	**0.0001**	0.0001	99.9985	99.9881	0.4451
R_z	OLS	-	-	7.0000	1.8758	0.2680	87.4581	71.3327	2.1736
	LLR	0.5615	0	8.7731	3.7841	0.4313	74.6997	53.8585	11.3711
	MRR2	0.3223	1.0000	2.9859	0.1023	0.0343	99.3157	96.3332	1.1210
	$MRR2_{rank}$	0.1995/0.2371	1.0000	1.7106	**0.0109**	**0.0064**	**99.9271**	**99.3178**	**0.3003**
	$MRR2_{+ve}$	0.2034/0.2338	1.0000	1.7976	0.0134	0.0075	99.9104	99.2022	0.3236
R_q	OLS	-	-	7.0000	1.1678	0.1668	91.0914	79.6376	1.3413
	LLR	0.4752	-	7.2006	1.6309	0.2265	87.5585	72.3543	12.1785
	MRR2	0.3410	1.0000	3.2302	0.2806	0.0869	97.8597	89.3986	1.9281
	$MRR2_{rank}$	0.2236/0.2309	1.0000	1.2258	**0.0059**	**0.0048**	**99.9552**	**99.4151**	**0.8300**
	$MRR2_{+ve}$	0.1779/0.2215	1.0000	1.6688	0.0249	0.0149	99.8099	98.1771	0.9705
R_t	OLS	-	-	7.0000	6.1460	0.8780	89.4368	75.8555	7.1019
	LLR	0.5015	-	7.7328	10.7671	1.3924	81.4945	61.7102	60.4051
	MRR2	0.3407	1.0000	3.2262	0.9555	0.2962	98.3578	91.8554	7.7502
	$MRR2_{rank}$	0.2617/0.2361	1.0000	1.6988	**0.0122**	**0.0072**	**99.9791**	**99.8030**	**2.5816**
	$MRR2_{+ve}$	0.1686/0.2011	1.0000	1.9499	0.0272	0.0139	99.9533	99.6166	2.6252

From Table 7, we observe that across the four responses, either $MRR2_{rank}$ or $MRR2_{+ve}$ gives best value of each statistics including PRESS** criterion which is key for the superior prediction of the values of the explanatory variables that would simultaneously optimize the four roughness parameters.

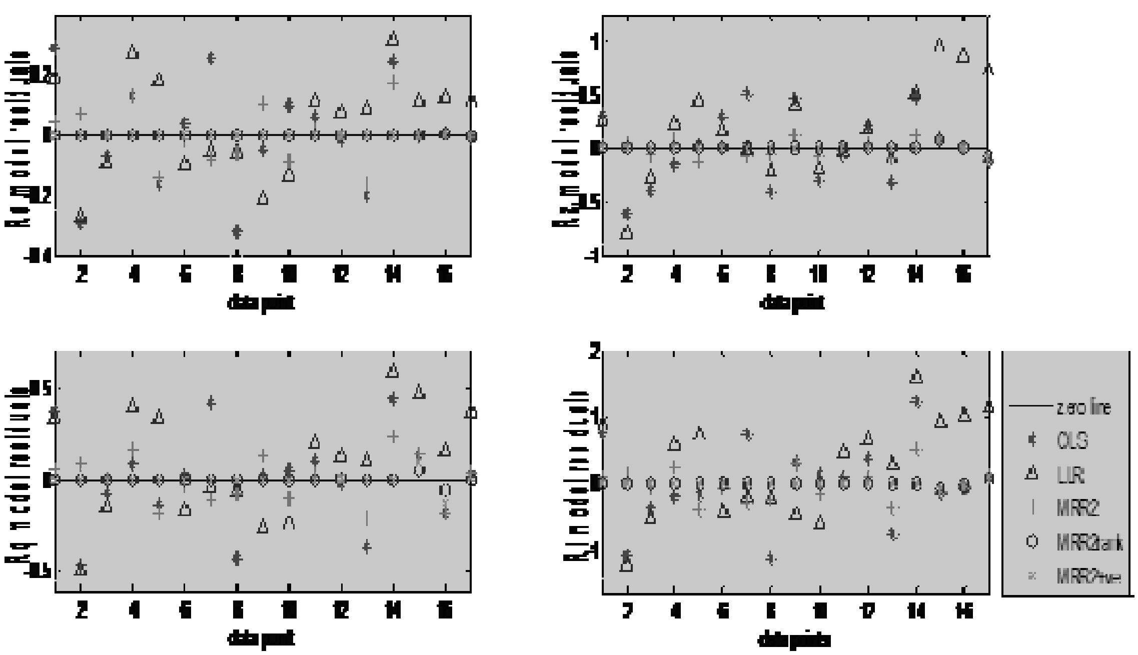

Fig. 4. Plots of residuals from each model for the surface roughness data.

The plots of the residual of each model in Figure4 shows that the residuals from $MRR2_{rank}$ and $MRR2_{+ve}$overlap at virtually all data points but are closest to the zero residual line across the four parameters of surface roughness. This indicates that $MRR2_{rank}$ and $MRR2_{+ve}$ fit the data comparatively better than OLS, LLR and the MRR2 that utilizes existing kernel weights.

Table 8. Optimization results based on desirability function for the surface roughness data.

MODEL	x_1	x_2	x_3	$\widehat{R}_a$	$\widehat{R}_z$	$\widehat{R}_q$	$\widehat{R}_t$	d_1	d_2	d_3	d_4	D(%)
OLS	0.1587	0.9497	0.8898	0.2280	0.1300	0.0310	6.1×10^{-8}	0.8299	0.9631	0.9870	1.0000	94.2
LLR	0.0804	0.9993	0.9999	2.8×10^{-7}	0.9483	0.0615	0.2926	1.0000	0.7117	0.9679	0.9224	89.3
MRR2	0.2402	0.9951	0.8421	0.1784	0.0227	1.5×10^{-6}	0.0002	0.8686	0.9961	1.0000	1.0000	96.4
$MRR2_{rank}$	0.2271	0.9599	0.8768	0.0113	5.2×10^{-6}	0.0457	0.0032	0.9990	1.0000	0.9778	1.0000	**99.4**
$MRR2_{+ve}$	0.2514	0.9920	0.8412	0.1066	0.1017	0.0043	1.7×10^{-5}	0.9246	0.9718	1.0000	1.0000	97.4

The results presented in Table 8 shows that $MRR2_{rank}$ provides the best optimal value of 99.4% given cutting speed of 0.2271 (**254.3979m/min**), feed rate of (**0.1774mm/rev**), and depth of cut of (**0.4388mm**). $MRR2_{+ve}$ came a close second with a optimal desirability value of 97.4%.

Minced fish data

The problem is from the literature. It involves three explanatory variables which include x_1(washing temperatures),x_2 (washing time),x_3 (washing ratio of water volume to sample weight and four responses variablesy_1, y_2, y_3 and $y_4$representing springiness, thiobarbituric acid number, (TBA), cooking loss, and whiteness index, respectively. A CCD was used in the in the experimental stage of the study.

According to the information presented in Shah *et al*. (2004), for each response, the model terms significant for the specified parametric models for OLS regression approach as well as the process requirements are as follows:

y_1**:** the intercept, x_1 and x_1^2;

Maximize $y_1$with lower bound L=1.70, and target value $\emptyset$= 1.92;

y_2: the intercept,x_1, x_2, x_1^2, and x_1x_2;

Minimize $y_2$with target value $\emptyset$ =20.16 and upper bound U=21.00;

y_3**:** the intercept, x_1, x_2, x_3, x_1^2, x_1x_2, x_1x_3, and x_3^2;

Minimize y_3 with target value $\emptyset$ =16.80, and upper bound U =20.00;

y_4**:** the intercept, x_1 and x_1^2.

Maximize y_4 with lower bound L =45.00, and target value $\emptyset$ =50.98.

The modeling spaces for LLR and LQR are based on the ones used for the OLS approach (Wan and Birch, 2011; Edionwe *et al*., 2014, Edionwe *et al*., 2016).

The data is presented in Table 9. The goodness of fit and optimization results based on the desirability function are presented in Table 10 and 11, respectively. The plots of residuals for each model are shown in Figure 5.

Table 9. Minced fish data.

i	Coded values			Response values			
	x_1	x_2	x_3	y_1	y_2	y_3	y_4
1	0.2030	0.2030	0.2030	1.83	29.31	29.50	50.36
2	0.7970	0.2030	0.2030	1.73	39.32	19.40	48.16
3	0.2030	0.7970	0.2030	1.85	25.16	25.70	50.72
4	0.7970	0.7970	0.2030	1.67	40.18	27.10	49.69
5	0.2030	0.2030	0.7970	1.86	29.82	21.40	50.09
6	0.7970	0.2030	0.7970	1.77	32.20	24.00	50.61
7	0.2030	0.7970	0.7970	1.88	22.01	19.60	50.36
8	0.7970	0.7970	0.7970	1.66	40.02	25.10	50.42
9	0.0000	0.5000	0.5000	1.81	33.00	24.20	29.31
10	1.0000	0.5000	0.5000	1.37	51.59	30.60	50.67
11	0.5000	0.0000	0.5000	1.85	20.35	20.90	48.75
12	0.5000	1.0000	0.5000	1.92	20.53	18.90	52.70
13	0.5000	0.5000	0.0000	1.88	23.85	23.00	50.19
14	0.5000	0.5000	1.0000	1.90	20.16	21.20	50.86
15	0.5000	0.5000	0.5000	1.89	21.72	18.50	50.84
16	0.5000	0.5000	0.5000	1.88	21.21	18.60	50.93
17	0.5000	0.5000	0.5000	1.87	21.55	16.80	50.98

Table 10. Goodness of fit of the regression models for the minced fish data.

Response	Model	b_1/b_2	λ	DF	SSE	MSE	R^2	R^2_{Adj}	$PRESS$**
y_1	**OLS**	-	-	14.0000	0.0231	0.0017	92.1256	91.0007	0.0042
	LLR	0.1463	-	12.1398	0.0126	0.0010	95.6990	94.3314	0.0026
	MRR2	0.1665	1.0000	12.2477	0.0126	0.0010	95.6938	94.3746	0.0026
	$MRR2_{rank}$	0.3322/0.2772	1.0000	10.2971	**0.0015**	**0.0001**	**99.4922**	**99.2110**	**0.0007**
	$MRR2_{+ve}$	0.2796/0.2747	1.0000	10.3231	0.0017	0.0002	99.4367	99.1270	0.0008
y_2	**OLS**	-	-	12.0000	90.9033	7.5753	93.3851	91.1801	19.6113
	LLR	0.4363	-	11.2152	45.3568	21.8771	82.1456	74.5284	36.4407
	MRR2	0.2567	0.5121	10.3049	57.4264	5.5727	95.8211	93.5117	16.6283
	$MRR2_{rank}$	0.3060/0.3397	1.0000	6.2143	3.0026	0.4832	99.7815	99.4374	7.0541
	$MRR2_{+ve}$	0.1948/0.3965	0.9581	6.2799	**1.1564**	**0.1841**	**99.9159**	**99.7856**	**7.0254**
y_3	**OLS**	-	-	9.0000	41.1338	4.5704	84.0607	71.6634	20.3074
	LLR	0.5371	-	8.3794	82.1622	9.8053	68.1622	39.2071	17.0573
	MRR2	0.3646	0.9063	4.4195	6.9603	1.5749	97.3029	90.2357	11.3852
	$MRR2_{rank}$	0.1502/1.0000	1.0000	2.9020	0.1942	0.0669	99.9248	99.5852	4.1416
	$MRR2_{+ve}$	0.1432/1.0000	1.0000	3.0175	**0.0825**	**0.0273**	**99.9680**	**99.8305**	**2.2142**
y_4	**OLS**	-	-	14.0000	98.8048	14.2003	54.1259	47.5724	48.9401
	LLR	0.1197	-	12.0308	12.2627	1.0193	97.1704	96.2369	**17.1477**
	MRR2	0.1218	1.0000	12.0366	12.2644	1.0189	97.1700	96.2381	19.1321
	$MRR2_{rank}$	0.4986/0.2568	1.0000	11.7465	7.1217	0.6063	98.3567	97.7616	18.7498
	$MRR2_{+ve}$	0.1569/0.1883	1.0000	11.0652	**5.2392**	**0.4735**	**98.7911**	**98.2519**	17.8245

A comparison of the results in Table 10 shows that, across the four responses, either $MRR2_{rank}$ or $MRR2_{+ve}$ gives best value of each statistics including PRESS** criterion (except for y_4) and very outstanding values of the coefficient of variation (R^2) and the adjusted coefficient of variation (R^2_{Adj}).

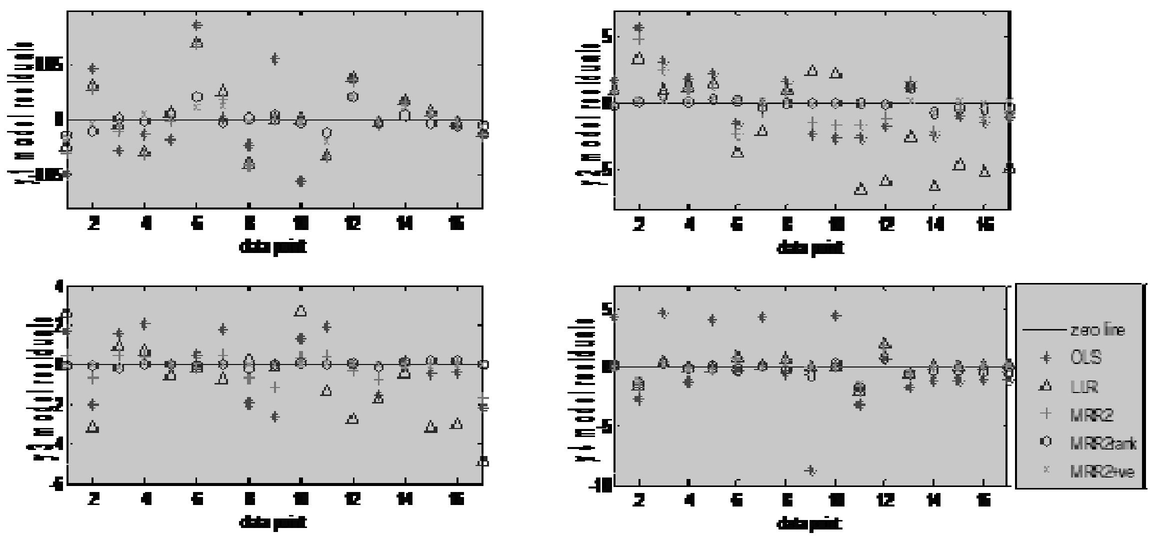

Fig. 5. Plots of model residuals for the minced fish data.

Again, the plots of the residuals of each model in Figure 5 reveals that those from both $MRR2_{rank}$ and $MRR2_{+ve}$ overlap in virtually at all data points but are closest to the zero residual line across the four parameters of surface roughness. This further indicates that both $MRR2_{rank}$ and $MRR2_{+ve}$ give comparatively better estimates of the data under study.

Table 11. Optimal results of the models based on desirability function for the minced fish data.

MODEL	x_1	x_1	x_3	$\hat{y}_1$	$\hat{y}_2$	$\hat{y}_3$	$\hat{y}_4$	d_1	d_2	d_3	d_4	$D(\%)$
OLS	0.3764	1.0	0.7155	1.9071	19.4993	17.2185	50.3018	0.9415	1.0000	0.8692	0.8866	92.3
LLR	-	-	-	-	-	-	-	-	-	-	-	0.0
MRR2	0.3583	1.0	0.6731	1.8965	18.9967	18.0381	51.4267	0.8933	1.0000	0.6131	1.0000	86.0
$\boldsymbol{MRR2_{rank}}$	0.3761	0.8438	0.6801	1.8939	19.5459	15.6267	51.2548	0.8816	1.0000	1.0000	1.0000	**96.9**
$\boldsymbol{MRR2_{+ve}}$	0.4763	1.0000	0.7846	1.8828	20.1610	16.7995	50.6137	0.8309	0.9989	1.0000	0.9387	94.0

From the results in Table 11, it is seen that the comparatively better goodness of fit of $MRR2_{rank}$ and $MRR2_{+ve}$ reflect in their ability to finding better settings of the explanatory variables that optimize the four responses according to the production requirement of the study. In particular, $MRR2_{rank}$gives the optimal setting of the explanatory variables that corresponds to the best desirability measure of **96.9%,** indicating a product that meets approximately **97%** of the production requirements. LLR gives a zero desirability measure because of the abysmally poor goodness of fit in y_2 and y_3.

Simulation data

The simulation study focuses on how each of the models performs when random errors with different variance, σ^2, are added to the responses generated from three different underlying or specified polynomial models. In other words, the essence of the simulation is to investigate how each of the regression model fares in the presence of varying degree of deviation from the correct but unknown polynomial depicted by f in equation (1).

Each simulation comprises 500 data sets, each of which is based on the following underlying models**:**

Underlying Model I:

$y_i = 25 - 13x_{i1} + 19x_{i1}^2 + \varepsilon_i,$

Underlying Model II:

$y_i = 42 + 12x_{i1} - 5x_{i2} - 15x_{i1}^2 - 7x_{i2}^2 + 8x_{i1}x_{i2} + \varepsilon_i,$

Underlying Model III:

$y_i = 57 - 11x_{i1} + 14x_{i2} + 10x_3 + 15x_{i1}^2 - 6x_{i2}^2 - 10x_{i3}^2 - 5x_{i1}x_{i2} - 12x_{i2}x_{i3} - 4x_{i3} + \varepsilon_i,$

where x_{i1}, x_{i2}, and x_{i3}, $i = 1,2,\dots,n$, are the respective values of the explanatory variables x_1, x_2 and x_3 from the coded values of the explanatory variables in Table 6 or Table 9, ε_i, $i = 1,2,\dots,n$, is the i^{th} value of the normally distributed random error, $\boldsymbol{\varepsilon}$ with mean 0 and variance $= \sigma^2$, that is $\boldsymbol{\varepsilon} \sim N(0, \sigma^2)$. For each of the 500 data sets from each of underlying models, σ^2 varies from 1, 4, 9, 16, 26 and 36.

In the modeling of RSM data, it is highly desirable that the fitted curve approximates the underlying model as precisely as possible (He *et al*., 2012). Because we know the true model in this study, the best criterion for comparing the performance or reliability of each regression model would be the average Sum of Squared of Errors (AVESSE) of the fitted response from the true response. The AVESSE represents how well each regression model estimates or predicts the true raw response generated by the underlying model.

Hence, the average SSE, (AVESSE), given by each regression model using each 500 data sets generated from each underlying model at different random error variance is presented in Table 12.

Table 12. AVESSE of each regression model for each underlying model of the simulated data.

Underlying Model	$\sigma^2(\varepsilon)$	OLS	LLR	MRR2	$MRR2_{rank}$	$MRR2_{+ve}$
Model I	1.0	1.1577	8.1479	1.0125	0.0630	**0.0455**
	4.0	4.7250	11.6464	4.3520	0.2453	**0.2318**
	9.0	10.5933	16.6931	8.7604	**0.5527**	0.6423
	16.0	18.7742	23.8695	16.1778	0.9896	**0.9118**
	25.0	29.9152	36.1598	26.3740	**1.4454**	1.4910
	36.0	42.0129	46.4201	37.9899	**2.0980**	2.6103
Model II	1.0	0.9102	7.7899	0.7173	**0.0454**	0.0555
	4.0	3.6423	10.2112	3.3339	0.2716	**0.2251**
	9.0	8.3273	14.5293	6.5257	**0.4012**	0.5127
	16.0	14.8887	21.5428	11.9975	**0.8586**	0.9764
	25.0	22.9982	28.5557	21.2238	1.5920	**1.4375**
	36.0	32.5948	37.6278	25.8134	2.4598	**2.1265**
Model III	1.0	0.5846	11.8118	0.3280	**0.0232**	0.0804
	4.0	2.2593	13.5938	1.2999	**0.2988**	0.3449
	9.0	5.2867	15.7548	2.7279	0.6588	**0.6284**
	16.0	9.4035	19.0534	5.3636	**1.0648**	1.2683
	25.0	14.4029	23.2839	7.3554	2.5551	**2.4341**
	36.0	21.0174	27.8234	10.6717	**2.8827**	3.5001

Again, the results from Table 12 reveals the best AVESSE for each underlying models across all the values of the random error variance are either given by $MRR2_{rank}$ or $MRR2_{+ve}$.
This confirms that the robustification of the kernel weights impact significantly on the performance of the MRR2 model.

CONCLUSION

In this study, we proposed the robustification of the Gaussian kernel weights that can be utilized by the LLR portion for improved performance of MRR2 and presented two different methods of transforming the residuals from the OLS component in order to achieve the robustification. Several data, including those generated from real life experiments involving multiple response surface roughness, were analyzed to validate the impact of the proposed robustification on the performance of MRR2. Comparisons of the overall performance of the MRR2 that utilizes the robustified kernel weights and that of its competitors (OLS, LLR and MRR2 that utilizes existing kernel weights) in terms of goodness of fit (Tables 7, 10 and 12), optimal solutions based on the desirability function (Tables 8 and 11) and plots of residuals from each regression model (Figures 4 and 5) clearly show that the robustified kernel weights significantly improves the fortunes of the MRR2. Specifically, the optimal cutting speed, feed rate and depth of cut found to be 254.3979m/min, 0.1774mm/rev and 0.4388mm, respectively, corresponding to an outstanding desirability of 99.4% will be of a very high practical relevance in the machining industries where mild steel (EN10) material is used. This finding will guarantee a more judicious use of material and machine tools and leads to improved returns to overall capital investments in the industries.

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